会议手册

数值方法青年研讨会

2022年5月13-14日，5月20-21日

国家天元数学东南中心

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组委会：

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一、日程表

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	09:10-09:50	**Some numerical issues regarding deep neural network approximations for PDEs** 周涛（中国科学院数学与系统科学研究院）	
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	10:00-10:40	**On orthogonality of Kohn-Sham wavefunctions in ground state calculations** 胡光辉（澳门大学）	许传炬
	10:40-11:20	**Implicitly linear Jacobi spectral-collocation methods for two-dimensional weakly singular Volterra-Hammerstein integral equations** 黄秋梅（北京工业大学）	
	11:20-12:00	**Implicit-explicit relaxation Runge-Kutta methods: construction, analysis and applications to PDEs** 李东方（华中科技大学）	
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	16:50-17:30	**Optimal convergence of arbitrary Lagrangian–Eulerian iso-parametric finite element methods for parabolic equations in an evolving domain** 李步扬（香港理工大学）	
	17:30-18:10	**Error estimate and numerical simulation for wormhole propagation** 李晓丽（山东大学）	

日期	时间	事项	主持人
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	09:10-09:50	**High-order multi-resolution central Hermite WENO schemes for hyperbolic conservation laws** 陶詹晶（吉林大学）	
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05月20日上午	08:30-09:10	**Accuracy-enhancement of discontinuous Galerkin methods for PDEs containing high order spatial derivatives** 徐岩（中国科学技术大学）	邱建贤
	09:10-09:50	**非线性问题的多重网格算法** 谢和虎（中国科学院数学与系统科学研究院）	
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	10:00-10:40	**Fluid Modeling in Three Dimensions: Topological Classification, Boolean Algebra, Geometric Representation, and Their Applications to Mean Curvature Flows** 张庆海（浙江大学）	陈黄鑫
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	11:20-12:00	**Unconditionally energy-decreasing high-order stabilized Implicit-Explicit Runge--Kutta methods for phase-field models** 杨将（南方科技大学）	

日期	时间	事项	主持人
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	14:40-15:20	**Recent progress on variable-time-step schemes for subdiffusion and diffusion equations** 张继伟（武汉大学）	
	15:20-16:00	**A vertex-centered diamond scheme for solving heterogeneous and anisotropic diffusion problems on arbitrary polygonal meshes** 张晓平（武汉大学）	
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	16:10-16:50	**Global well-posedness and exponential stability for Maxwell equations under delayed boundary conditions in metamaterials** 姚昌辉（郑州大学）	杜魁
	16:50-17:30	**Recent development of numerical analysis for the Landau-Lifshitz equation** 安荣（温州大学）	
	17:30-18:10	**Machine learning based numerical methods for multiscale problems** 张镭（上海交通大学）	

05月21日上午	08:30-09:10	**An ensemble algorithm for two-domain couple random parabolic PDEs** 郑海标（华东师范大学）	黄文
	09:10-09:50	**Finite element discretization and fast solver for problems in non-divergence form with Cordes coefficients** 吴朔男（北京大学）	
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	11:20-12:00	**Diffusion Coefficient Identification for Elliptic and Parabolic Problems** 周知（香港理工大学）	

二、报告题目与摘要

Nucleation of Quasicrystals

张磊（北京大学）

Despite the fact that tremendous efforts have been made on the study of quasicrystals since their discovery in 1984, nucleation of quasicrystals, viz. the emergence of a quasicrystal from a crystalline phase, still presents an unsolved and challenging problem. The difficulties lie in that quasicrystals and crystals are incommensurate structures in general, so there are no obvious epitaxial relations between them. In this talk, we proposed a saddle dynamics method to solve this problem by applying the Landau theory of phase transitions. We obtained the accurate critical nuclei and transition pathways connecting crystalline and quasicrystalline phases. The results reveal that phase transitions between the crystalline and quasicrystalline phases could follow two possible pathways, corresponding to a one-stage phase transition and a two-stage phase transition involving a metastable lamellar quasicrystalline state, respectively. The proposed computational methodology not only reveals the mechanism of nucleation of quasicrystals, but also paves the way to investigate a wide range of physical problems undergoing the first-order phase transitions.

Some numerical issues regarding deep neural network approximations for PDEs

周涛（中国科学院数学与系统科学研究院）

Deep neural networks have been widely used for solving PDEs in recently years. In this talk, we shall discuss some numerical issues for such approaches. In particular, we shall present some recent ideas for dealing with nonlocal operators and effective sampling strategies on unbounded domains.

On orthogonality of Kohn-Sham wavefunctions in ground state calculations

胡光辉（澳门大学）

Orthogonality of wavefunctions in Kohn-Sham density functional theory brings a bottleneck of numerical simulations of ground states. In this talk, KSDFT will be briefly introduced, and our several recent works towards resolving this efficiency bottleneck will be described in detail. Further, methods for accelerating simulations such as adaptive mesh methods will also be discussed, as well as related numerical issues.

Implicitly linear Jacobi spectral-collocation methods for two-dimensional weakly singular Volterra-Hammerstein integral equations

黄秋梅（北京工业大学）

Weakly singular Volterra integral equations of the second kind typically have nonsmooth solutions near the initial point of the interval of integration, which seriously affects the accuracy of spectral methods. We present Jacobi spectral-collocation method to solve two-dimensional weakly singular Volterra- Hammerstein integral equations based on smoothing transformation and implicitly linear method. The solution of the smoothed equation is much smoother than the original one after smoothing transformation and the spectral method can be used. For the Hammerstein nonlinear term, the implicitly linear method is applied to simplify the calculation and improve the accuracy. Convergence analysis in the L∞−norm is carried out and the exponential convergence rate is obtained. Finally, we demonstrate the efficiency of the proposed method by numerical examples.

Implicit-explicit relaxation Runge-Kutta methods: construction, analysis and applications to PDEs

李东方（华中科技大学）

Spatial discretizations of time-dependent partial differential equations usually result in a large system of semi-linear and stiff ordinary differential equations. Taking the structures into account, we develop a family of linearly implicit and high order accurate schemes for the time discretization, using the idea of implicit-explicit Runge-Kutta methods and the relaxation techniques. The proposed schemes are monotonicity preserving/conservative for the original problems, while the previous linearized methods are usually not. We also discuss the linear stability and strong stability preserving (SSP) property of the new relaxation methods. Numerical experiments on several typical models are presented to confirm the effectiveness of the proposed methods.

An efficient iterative method for solving multiple scattering problem

汪波（湖南师范大学）

An efficient iterative method is proposed for solving multiple scattering problem in locally inhomogeneous media. The key idea is to enclose the inhomogeneity of the media by well separated artificial boundaries and then apply purely outgoing wave decomposition for the scattering field outside the enclosed region. As a result, the original multiple scattering problem can be decomposed into a finite number of single scattering problems, where each of them communicates with the other scattering problems only through its surrounding artificial boundary. Accordingly, they can be solved in a parallel manner at each iteration. This framework enjoys a great flexibility in using different combinations of iterative algorithms and single scattering problem solvers. The spectral element method seamlessly integrated with the non-reflecting boundary condition and the GMRES iteration is advocated and implemented in this work. The convergence of the proposed method is proved by using the compactness of involved integral operators. Ample numerical examples are presented to show its high accuracy and efficiency.

Multiscale modeling and simulation of additive manufacturing based on fused deposition technique

李义宝（西安交通大学）

The issue of multiscale modeling has received considerable critical attention for additive manufacturing with fused deposition technique, which involves complex physicochemical phase transitions and thermodynamic behaviors. Lack of multiscale theoretical model poses significant challenges for prediction in AM processes driven by the rapidly evolving temperature field, including the nonuniformity of tracks, spheroidization effect of materials and intertrack voids. Few studies have systematically investigated the mapping relationship between physical environment and virtual environment. Inspired by the experimental results, we will develop a multi--scale system to describe the FD process in AM, which is coupled the conductive heat transfer model and the dendritic solidification model. The simulation requires a computational framework with high performance because of the cumulative effect of heat transfer between different layers. The proposed system is capable of simulating the material state with proper parameter at macro- and micro-scale and directly used to capture multiple physical phenomena. We further carry out several numerical experiments and comparative experiments to verify the robustness and applicability of the proposed model.

超导模型中 Ginzburg-Landau 方程的最低价混合元的误差分析

高华东（华中科技大学）

This talk is concerned with new error analysis of a lowest-order backward Euler Galerkin-mixed finite element method for the time-dependent Ginzburg--Landau equations. The method is based on a commonly-used non-uniform approximations, in which linear Lagrange element, the lowest order Nedelec edge element and Raviart--Thomas face element are used for the order parameter ψ, the magnetic field $curl \mathbf{A}$ and the magnetic potential $\mathbf{A}$, respectively. This mixed method has been widely used in practical simulations due to its low cost and ease of implementation. In the Ginzburg-Landau model, the order parameter ψ is the most important variable, which indicates the state of the superconductor. An important feature of the method is the inconsistency of the approximation orders. A crucial question is how the first-order approximation of $(curl \mathbf{A}, \mathbf{A})$ influences the accuracy of ψ_h. The main purpose is to establish the second-order accuracy for the order parameter in spatial direction, although the accuracy for $curl \mathbf{A}, \mathbf{A}$ is in the first order only. Previous analysis only gave the first order convergence for all three variables due to certain artificial pollution involved in analysis. Our analysis is based on a nonstandard quasi-projection for ψ and the corresponding more precise estimates, including in H^{-1}-norm. With the quasi-projection, we prove that the lower-order approximation to $curl \mathbf{A}, \mathbf{A}$ does not pollute the accuracy of ψ_h. Our numerical experiments confirm the optimal convergence of ψ_h. The approach can be extended to many other multi-physics models.

Numerical analysis of deep solvers for the second order elliptic differential equations

吕锡亮（武汉大学）

In recent years, deep neural networks (DNN) have been shown to be a powerful tool for solving PDEs empirically. However, numerical analysis of these Deep solvers are far from complete. In this work, we provide the convergence rate to two typical deep solvers: DRM and PINNs for the second order elliptic equations. The error estimations are decomposed into approximation error and statistical error, which depend on the number of training samples, depth and width of the deep neural networks.

Optimal convergence of arbitrary Lagrangian–Eulerian iso-parametric finite element methods for parabolic equations in an evolving domain

李步扬（香港理工大学）

An optimal-order error estimate is presented for the arbitrary Lagrangian–Eulerian (ALE) finite element method for a parabolic equation in an evolving domain, using high-order iso-parametric finite elements with flat simplices in the interior of the domain. The mesh velocity can be a linear approximation of a given bulk velocity field or a numerical solution of the Laplace equation with specified boundary value matching the velocity of the boundary. The optimal order of convergence is obtained by comparing the numerical solution with the ALE-Ritz projection of the exact solution, and by establishing an optimal-order estimate for the material derivative of the ALE-Ritz projection error.

Error estimate and numerical simulation for wormhole propagation

李晓丽（山东大学）

In this talk, we shall first construct the decoupled fully discrete scheme for wormhole model with the Darcy–Brinkman–Forchheimer framework. Error estimates for the pressure, velocity, porosity, concentration and auxiliary flux with second-order superconvergence in different discrete norms are established rigorously and carefully on non-uniform grids. Next we will discuss the linear finite difference scheme on staggered grids for wormhole propagation with heat transmission process. Optimal error estimates for the velocity, pressure, concentration, porosity and temperature in different norms are established by establishing several auxiliary lemmas for the highly coupled nonlinear system. We also give some recent progress for this model in cylindrical coordinates. Numerical experiments in two- and three-dimensional cases are provided to verify our theoretical results and illustrate the capabilities of the constructed method.

On the Strang splitting methods for Allen-Cahn equations

权超禺（南方科技大学）

We consider a class of second-order Strang splitting methods for Allen-Cahn equations with polynomial or logarithmic nonlinearities. For the polynomial case both the linear and the nonlinear propagators are computed explicitly. We show that this type of Strang splitting scheme is unconditionally stable regardless of the time step. Moreover we establish strict

energy dissipation for a judiciously modified energy which coincides with the classical energy up to $\mathcal O(\tau)$ where τ is the time step. For the logarithmic potential case, since the continuous-time nonlinear propagator no longer enjoys explicit analytic treatments, we employ a second order in time two-stage implicit Runge--Kutta (RK) nonlinear propagator together with an efficient Newton iterative solver. We prove a maximum principle which ensures phase separation and establish energy dissipation law under mild restrictions on the time step. These appear to be the first rigorous results on the energy dissipation of Strang-type splitting methods for Allen-Cahn equations.

High-order multi-resolution central Hermite WENO schemes for hyperbolic conservation laws

陶詹晶（吉林大学）

In this paper, a class of high-order multi-resolution central Hermite WENO (C-HWENO) schemes for solving one- and two-dimensional hyperbolic conservation laws is proposed. Formulated in a central finite volume framework on staggered mesh, the methods adopt the multi-resolution HWENO reconstructions in space and the natural continuous extension of Runge-Kutta methods in time. The proposed methods are based on the zeroth-order and first-order moments of the solution defined on a series of hierarchical central spatial stencils. The linear weights of such HWENO reconstructions can be any positive numbers as long as their sum equals one, which leads to much simpler implementation and better cost efficiency. Furthermore, the first-order moments in the troubled-cell are modified to enhance the resolution of the methods. Meanwhile, our new methods have compact stencils in the reconstructions and requires neither numerical fluxes nor flux splitting. Extensive one- and two-dimensional numerical examples are performed to illustrate the accuracy and high resolution of the new C-HWENO schemes.

Data-driven modeling for nonlinear dynamic systems

姜立建（同济大学）

Data-driven discovery has become a very important paradigm for science and engineering. In this talk, I will present data-driven modeling for nonlinear dynamic systems using Koopman operator. With time-series data, Koopman operator is used to model deterministic dynamic systems and stochastic dynamic systems. The resultant data-driven models can be used to make state forecast and quantify models uncertainty.

An efficient unconditionally stable method for computing Dirichlet partitions in arbitrary domains

王东（香港中文大学（深圳））

A Dirichlet k-partition of a domain is a collection of k pairwise disjoint open subsets such that the sum of their first Laplace--Dirichlet eigenvalues is minimal. In this talk, we propose a new relaxation of the problem by introducing auxiliary indicator functions of domains and develop a simple and efficient diffusion generated method to compute Dirichlet k-partitions for

arbitrary domains. The method only alternates three steps: 1. convolution, 2. thresholding, and 3. projection. The method is simple, easy to implement, insensitive to initial guesses and can be effectively applied to arbitrary domains without any special discretization. At each iteration, the computational complexity is linear in the discretization of the computational domain. Moreover, we theoretically prove the energy decaying property of the method. Experiments are performed to show the accuracy of approximation, efficiency, and unconditional stability of the algorithm. We apply the proposed algorithms on both 2- and 3-dimensional flat tori, triangle, square, pentagon, hexagon, disk, three-fold star, five-fold star, cube, ball, and tetrahedron domains to compute Dirichlet k-partitions for different k to show the effectiveness of the proposed method. Compared to previous work with reported computational time, the proposed method achieves hundreds of times acceleration.

A linearly implicit finite element full-discretization scheme for SPDEs with non-globally Lipschitz coefficients

王小捷（中南大学）

This talk is mainly concerned with strong approximations of parabolic stochastic partial differential equations (SPDEs) with polynomially growing nonlinearity. Unlike the globally Lipschitz case, the usual linearly implicit time-stepping scheme for the considered problem leads to divergent approximations. Based on a spatial discretization of the continuous problem, a tamed version of linearly implicit Euler type time-steeping scheme is proposed and the resulting approximation errors are analyzed in a strong sense. Optimal strong convergence rates are proved under mild regularity assumptions.

Accuracy-enhancement of discontinuous Galerkin methods for PDEs containing high order spatial derivatives

徐岩（中国科学技术大学）

In this paper, we consider the accuracy-enhancement of discontinuous Galerkin (DG) methods for solving partial differential equations (PDEs) with high order spatial derivatives. It is well known that there are highly oscillatory errors for finite element approximations to PDEs that contain hidden superconvergence points. To exploit this information, a Smoothness-Increasing Accuracy-Conserving (SIAC) filter is used to create a superconvergence filtered solution. This is accomplished by convolving the DG approximation against a B-spline kernel. Previous theoretical results about this technique concentrated on first- and second-order equations. However, for linear higher order equations, Yan and Shu numerically demonstrated that it is possible to improve the accuracy order to $2k+1$ for local discontinuous Galerkin (LDG) solutions using the SIAC filter. In this work, we firstly provide theoretical proof for this observation. Furthermore, we prove the accuracy order of the ultra-weak local discontinuous Galerkin (UWLDG) solution could be improved to $2k+2-m$ using the SIAC filter, where $m=[\frac{n}{2}]$, n is the order of PDEs. Finally, we computationally demonstrate that for nonlinear higher order PDEs, we can also obtain a superconvergence approximation with the accuracy order of $2k+1$ or $2k+2-m$ by convolving the LDG solution and the UWLDG solution against the SIAC filter, respectively.

非线性问题的多重网格算法

谢和虎（中国科学院数学与系统科学研究院）

多重网格算法虽然已经有几十年的发展历史了，但基本的框架都是针对线性问题来设计的。当应用于求解非线性问题时，一般的方式是先进行线性化，然后使用多重网格方法求解其中的线性问题。这样的方式通常称为内、外迭代。多重网格的应用可以使得内迭代的效率达到最优，但最终的计算量同时也取决于外迭代的次数。对于一般的非线性方程，外迭代次数往往难于估计，这样就使得多重网格求解非线性方程的最终计算量往往依赖于问题的非线性强度，即多重网格的使用并没有改变非线性迭代本身。

本报告中，我们通过应用定义在粗网格上的有限元空间来构造一个求解非线性问题的扩展子空间，同时结合张量组装技术使得求解非线性问题的渐近计算量变得与非线性迭代次数无关而达到绝对最优。 同时我们也将介绍这种思想在一些问题中的应用。

Fluid Modeling in Three Dimensions: Topological Classification, Boolean Algebra, Geometric Representation, and Their Applications to Mean Curvature Flows

张庆海（浙江大学）

Solid modeling has been an established field while fluid modeling is still an undefined concept. However, there is a need of fluid modeling in numerically solving PDEs on complex moving domains: sometimes we have to have a solid ground for the topology and geometry of complex domains so that subsequent analysis can be performed. We answer this need by proposing for three-dimensional continua a complete topological classification, a Boolean algebra, and a geometric representation of their boundary. We united these techniques in a framework for numerically solving geometric PDEs such as the mean curvature flows with fourth- and higher-order convergence.

A variational analysis for a moving finite element method by using the Onsager principle

许现民（中国科学院数学与系统科学研究院）

In this talk, we will present an application the Onsager variational principle on numerical analysis for moving finite element method (MFEM). We give a novel derivation for the MFEM for gradient flow equations. We show that the discretized problem has the same energy dissipation structure as the continuous one. This enables us to do numerical analysis for the stationary solution of a nonlinear reaction diffusion equation using the approximation theory of free-knot piecewise polynomials. We show that under certain conditions the solution obtained by the moving finite element method converges to a local minimizer of the total energy when time goes to infinity. The global minimizer, once it is detected by the discrete scheme, approximates the continuous stationary solution in optimal order. For the dynamical problem, we present some proof for the convergence of the method by using the gradient flow theory in metric spaces.

Unconditionally energy-decreasing high-order stabilized Implicit-Explicit Runge--Kutta methods for phase-field models

杨将（南方科技大学）

Phase field models attract much attention these years. The energy naturally decreases along the direction of gradient flows, so it is rather significant for numerical methods to preserve this intrinsic structure. In order to guarantee the energy dissipation, various numerical schemes have been developed and among them, a simple but vital approach is implicit-explicit (IMEX) Runge--Kutta (RK) method. In this paper we prove that a class of high-order stabilized IMEX RK schemes unconditionally preserve the energy dissipation law for phase-field models with Lipschitz nonlinearity. A simple systematic condition is established to determine the nonlinear energy decaying property. This is the first work to prove that a high-order linear scheme can guarantee the dissipation of the original energy unconditionally. We also obtain the error estimate to show the convergence and accuracy. In the end, we give some concrete IMEX RK schemes as examples.

几类偏微分方程的两水平法

钟柳强（华南师范大学）

首先，针对非对称或不定二阶椭圆方程边值问题的协调有限元离散系统，给出了经典两网格法的 L2 误差估计，并设计和分析了一种改进两网格法. 其次，针对非对称或不定二阶椭圆方程边值问题的协调有限元离散系统，设计和分析了基于同一网格但不同次数的协调有限元空间的两水平和改进两水平法. 最后，针对二阶半线性椭圆问题边值问题的 SIPDG 离散系统，设计和分析了一种迭代两网格算法. 相应的数值实验验证了上述算法的有效性。

Recent progress on variable-time-step schemes for subdiffusion and diffusion equations

张继伟（武汉大学）

This talk focuses on the numerical analysis of reaction-subdiffusion equations with variable time step by taking the widely used L1 scheme for an example. For the stability analysis, the discrete complementary convolution (DCC) kernels are introduced to prove the discrete fractional-type Gronwall inequality. For the convergence analysis, the goals are theoretically challenging because the numerical Caputo formula always has a form of discrete convolutional summation. To circumvent this difficulty, an error convolution structure (ECS) analysis is proposed to express the consistence error for the discrete Caputo formula, which can significantly reduce consistence analysis for general nonuniform time steps. In addition, the technique here is also useful to extend the knowledge to study multi-step schemes such as BDF2 with variable time step for classical parabolic equations.

A vertex-centered diamond scheme for solving heterogeneous and anisotropic diffusion problems on arbitrary polygonal meshes

张晓平（武汉大学）

In this work, we propose a new vertex-centered diamond scheme for solving heterogeneous

and anisotropic diffusion problems, which is applicable for arbitrary polygonal meshes. The key to the construction of proposed scheme is how to approximate the solution function and its gradients. To this end, we use the piecewise constant function with some techniques. Under certain mesh regularities, we prove the discrete Poicare inequality, following by numerical stabilities and existence and uniqueness of the approximate solution. Furthermore, we investigate the H1 and L2 error estimate under the framework of the gradient discretization. Finally, a series of numerical examples show the good numerical performance of the scheme for diffusion equation with different diffusion tensor on different meshes.

Global well-posedness and energy dissipation estimation for Maxwell equations under delayed boundary conditions in metamaterials

姚昌辉（郑州大学）

In this paper, we concentrate on the model of Maxwell's system in metamaterials with a nonlinear boundary condition, which has considered two partial differential equations of the currents in the free electron related to the polarization and magnetization in metamaterials. Then we attempt to provide a further study for the double convolution of Maxwell's equations on time domain by Viscoelasticity Theory. Such a new initial boundary value problem is under the assumption of electrical sources and resistance effects are absent. Next the equations are transformed to an abstract nonlinear evolution equation. We proved the existence and uniqueness of solution with respect to the evolution equation by using nonlinear semigroup theory, which mainly explains the existence theory of nonlinear initial boundary value problem through the description of the basic theory of maximal monotone operators in reflexive Banach space. This provides a theoretical foundation for further research. Finally we give the exponential stability of energy with general nonlinear feedbacks on the whole domain.

Recent development of numerical analysis for the Landau-Lifshitz equation

安荣（温州大学）

The Landau-Lifshitz equation has been widely used to describe the dynamics of magnetization in a ferromagnetic material, which is highly nonlinear with the nonconvex constraint |m| =1. In designing numerical algorithm, a crucial issue is how to preserve the nonconvex constraint in the fully discrete level. A simple and frequently-used one is the sphere-projection method which projects the numerical solution onto a unit sphere at each time step. Due to the simplicity of the sphere-projection approach, the method has been extensively used in various applications. However, no rigorous error estimate is available up to now. In this talk, I will present an overview of numerical methods for the Landau-Lifshitz type equation and report recent development on numerical analysis of the sphere-projection method.

Machine learning based numerical methods for multiscale problems

张镭（上海交通大学）

In this talk, I will introduce our recent work on the machine learning based numerical

methods for multiscale problems. We will start with deep neural network based solver for multiscale elliptic equations, also a transformer based encoder-decoder model for multiscale operator learning. If time allowed, I will also introduce a recent joint work, DeePN^2: A deep learning-based non-Newtonian hydrodynamic model.

An ensemble algorithm for two-domain couple random parabolic PDEs

郑海标（华东师范大学）

In this talk, we propose an efficient ensemble algorithm for fast solving the two-domain couple random parabolic PDEs. We utilize the Monte Carlo method for the coupled model with random inputs to derive some deterministic two-domain numerical models, and use the ensemble technique to realize the fast computation of multiple problems. This ensemble algorithm employs the same coefficient matrix for all ensemble members at each time step to solve the linear systems with multiple right-hand sides. Moreover, it decouples the couple system into two subdomain problems, which can be solved in parallel. Theoretically, we prove that this algorithm is unconditionally stable.

Finite element discretization and fast solver for problems in non-divergence form with Cordes coefficients

吴朔男（北京大学）

In this talk, we first propose the C^0 finite element approximation of the Hamilton—Jacobi--Bellman (HJB) equations with Cordes coefficients. Using a discrete analog of the Miranda-Talenti estimate, we construct a family of non-standard finite element methods which does not have any artificial parameter. As a main feature of the proposed methods, the monotonicity constant of the HJB equations is preserved exactly at the discrete level, leading to a good performance for near-singular problems. As an extension, the discretization for the oblique derivative problem, which arises from transport boundary conditions for the Monge-Ampére equation, will be discussed.

In the semi-smooth steps, the linearized systems have large condition numbers, which depend not only on the mesh size but also on the parameters in the Cordes condition. We then design and analyze the auxiliary space preconditioners for the linearized systems. Both the additive and multiplicative preconditioners are shown to converge uniformly in the sense that the resulting condition number is independent of both the number of degrees of freedom and the parameter λ in the Cordes condition.

High order finite difference WENO methods with unequal-sized sub-stencils for the DP type equations

仲杏慧（浙江大学）

In this talk, we present finite difference weighted essentially non-oscillatory (WENO) schemes with unequal-sized sub-stencils for solving the Degasperis-Procesi (DP) and μ-Degasperis-Procesi (μDP) equations, which contain nonlinear high order derivatives, and possibly peakon solutions or shock waves. By introducing auxiliary variable(s), we rewrite

the DP equation as a hyperbolic-elliptic system, and the μ DP equation as a first order system. Then we choose a linear finite difference scheme with suitable order of accuracy for the auxiliary variable(s), and finite difference WENO schemes with unequal-sized sub-stencils for the primal variable. Comparing with the classical WENO scheme which uses several small stencils of the same size to make up a big stencil, WENO schemes with unequal-sized sub-stencils are simple in the choice of the stencil and enjoy the freedom of arbitrary positive linear weights. Another advantage is that the final reconstructed polynomial on the target cell is a polynomial of the same degree as the polynomial over the big stencil, while the classical finite difference WENO reconstruction can only be obtained for specific points inside the target interval. Numerical tests are provided to demonstrate the high order accuracy and non-oscillatory properties of the proposed schemes.

Eulerian-Lagrangian Runge-Kutta discontinuous Galerkin method for transport simulations

蔡晓峰（北京师范大学–香港浸会大学联合国际学院）

Semi-Lagrangian (SL) approach is attractive in transport simulations, e.g. in climate modeling and kinetic models, due to its numerical stability in allowing extra-large time-stepping sizes. For practical problems with complex geometry, schemes on the unstructured meshes are preferred. However, accurate and mass conservative SL methods on unstructured meshes are still under development and encounter several challenges. For instance, when tracking characteristics backward in time, high order curves are required to accurately approximate the shape of upstream cells, which brings in extra computational complexity. To avoid such computational complexity, we propose an Eulerian-Lagrangian Runge-Kutta discontinuous Galerkin method with discussion on the treatment of inflow boundary condition. The nonlinear WENO limiter is applied to control oscillations. Desired properties of the proposed method are numerically verified by a set of benchmarks tests.

Diffusion Coefficient Identification for Elliptic and Parabolic Problems

周知（香港理工大学）

Over the last few decades, inverse problems for PDEs have attracted substantial attention among mathematicians and engineers, due to their numerous important applications. These inverse problems are often mathematically and numerically more challenging compared with the corresponding direct problems, due to a lack of conventional well-posedness. Much work is still needed in developing efficient and accurate inversion algorithms with provable error bounds. In this talk, I will describe recent results in this aspect, illustrate with the inverse problem of recovering the diffusion coefficient in elliptic and parabolic equations from the distributed observation.

Tianyuan Mathematical Center in Southeast China

Tianyuan Mathematical Center in Southeast China (TMSE) is one of the five national mathematical centers approved and supported by the Tianyuan Mathematics Fund of the National Natural Science Foundation of China. The unveiling ceremony of the Center was held on January 8th 2019, which marks its official launch and operation.

Tianyuan Mathematics Fund was set up in 1990 with the aim of building China into a strong country in mathematics. It is in 2017 that the academic leadership committee of the Tianyuan Mathematics Fund of the National Natural Science Foundation of China launched the programme of establishing Tianyuan Mathematical Centers for balanced regional development of mathematics. The programme focuses on providing platforms for collaboration and research, and aims to enhance the research in relevant fields, foster research strengths and promote the progress of mathematical sciences. So far there are five Tianyuan Mathematical Centers which are respectively located in the southwest, northwest, northeast, southeast and central of China.

TMSE is based at Xiamen University and co-supported by several other universities in Fujian Province, Zhejiang Province, Guangdong Province, Jiangxi Province and Hainan Province. Centering upon the research on pure mathematics and its interdisciplinary application, TMSE will organize a variety of academic activities with a view to pooling high-caliber talents, promoting international cooperation, cultivating young mathematicians and ultimately advance the progress of mathematical sciences in the southeast part of China.

Under the guidance of the academic leadership committee of the Tianyuan Mathematics Fund of the National Natural Science Foundation of China and with the great support of Xiamen University and the joint efforts of its partner universities for TMSE, the center will make great strides in fostering first-class mathematical talents, producing world-class research and developing into a world-renowned platform for talent cultivation, joint research and academic cooperation.

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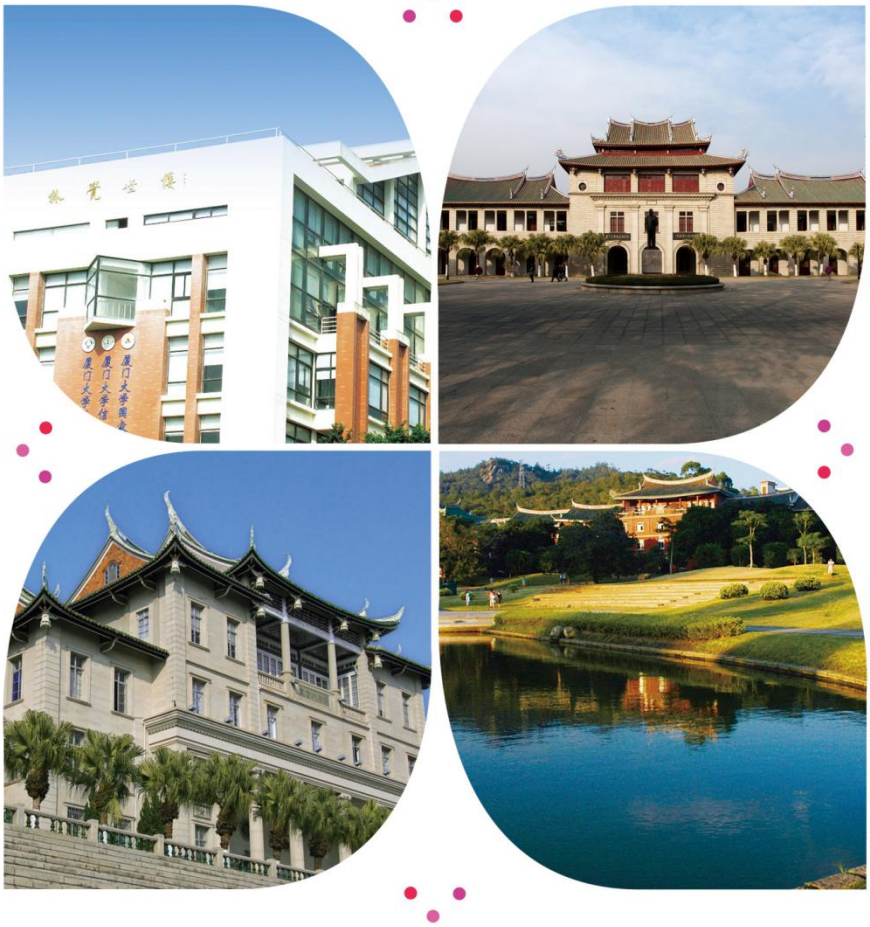

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