

# **Titles and Abstracts**

## **Recent developments in Ricci flow**

Richard Bamler (University of California, Berkeley)

**Abstract:** Ricci flows are a powerful geometric-analytical tool, as they have been used to prove important results in low-dimensional topology. In the first part of this talk I will focus on Ricci flows in dimension 3. I will briefly review Perelman's construction of Ricci flow with surgery, which led to the resolution of the Poincare and Geometrization Conjectures. Then I will discuss recent work of Lott, Kleiner and myself on an improved version of this flow, called "singular Ricci flow". This work allowed us to resolve the Generalized Smale Conjecture, concerning diffeomorphism groups, and a conjecture concerning the contractibility of the space of positive scalar curvature metrics on 3-manifolds. In the second part of the talk, I will focus on Ricci flows in higher dimensions. I will present a new compactness theory, which can be used to study the singularity formation of the flow, as well as its long-time asymptotics. I will discuss these and some further consequences. I will also convey some intuition of the new terminology that had to be introduced in connection with this compactness theory.

## **On bi-Lipschitz equivalence of a class of non-conformally flat spheres**

Sun-Yung Alice Chang (Princeton University)

**Abstract:** This is a report of some recent joint work with Eden Prywes and Paul Yang. The main result is a bi-Lipschitz equivalence of a class of metrics on 4-sphere under curvature constraints. The proof involves two steps: first a construction of quasi-conformal maps between two conformally related metrics in a positive Yamabe class, followed by the step of applying the Ricci flow to establish the bi-Lipschitz equivalence from such a conformal class to the standard conformal class on 4-sphere.

# **Symplectic vortex, its generalization, and Quantum Kirwan morphisms**

Bohui Chen (Sichuan University)

**Abstract:** Let  $(M, \omega, G, \mu)$  be a symplectic manifold with a Hamiltonian action. Let  $X$  be its symplectic reduction. The symplectic vortices on  $M$  were introduced by Salamon, Mundet i Riera and etc 20 years ago. It is used to construct the so-called Hamiltonian Gromov-Witten invariants. Essentially, this is a new type of the Gromov-Witten theory for the reduction  $X$  using the equivariant topological data of  $M$ . In this talk, I will review the topic following this line with  $L^2$ -moduli spaces of symplectic vortices. Furthermore, we generalize the vortex equation and introduce a new equivariant moduli space to give an equivariant Gromov-Witten theory for  $M$  when  $G$  is abelian. Combine these constructions, we may realize a quantum version of Kirwan map. The talk is mainly based on the joint work with Bai-Ling Wang and Rui Wang.

## **Improved Sobolev trace inequalities under constraints in the unit ball**

Xuezhang Chen (Nanjing University)

**Abstract:** Motivated by recent works of Alice Chang-Fengbo Hang and Fengbo Hang-Xiaodong Wang, we study the Sobolev trace inequalities on the Euclidean unit ball for higher order moments of the boundary volume element, and construct precise test functions to show that such inequalities are almost optimal. This generalizes the Aubin's improved Sobolev trace inequality on the unit ball for the vanishing of the first order moment case, which is closely related to the prescribed boundary mean curvature problem on the unit ball. This is joint with Mr. Nan Wu.

## Limiting of a class of Hermitian-Yang-Mills metrics

Jixiang Fu (Fudan University)

**Abstract:** I will talk about the limiting behavior of a family of Hermitian-Yang-Mills metrics on a class of rank two slope stable vector bundle over a product of two elliptic curves with Kahler metrics which approach a large Kahler metric limit.

## Integral-Einstein hypersurfaces and Simons-type inequalities in spheres

Jianquan Ge (Beijing Normal University)

**Abstract:** We introduce a generalization, the so-called Integral-Einstein (IE) submanifolds, of Einstein manifolds by combining intrinsic and extrinsic invariants of submanifolds in Euclidean spaces, in particular, IE hypersurfaces in unit spheres. A Takahashi-type theorem is established to characterize minimal hypersurfaces with constant scalar curvature (CSC) in unit spheres, which is the main object of the Chern conjecture: such hypersurfaces are isoparametric. For these hypersurfaces, we obtain some integral inequalities with the bounds characterizing exactly the totally geodesic hypersphere, the non-IE minimal Clifford torus  $S^1(\sqrt{\frac{1}{n}}) \times S^{n-1}(\sqrt{\frac{n-1}{n}})$  and the IE minimal CSC hypersurfaces. Moreover, if further the third mean curvature is constant, then it is an IE hypersurface or an isoparametric hypersurface with  $g \leq 2$  principal curvatures. In particular, all the minimal isoparametric hypersurfaces with  $g \geq 3$  principal curvatures are IE hypersurfaces. As applications, we also obtain some spherical Bernstein theorems. A universal lower bound for the average of squared length of second fundamental form of non-totally geodesic minimal hypersurface in unit spheres is established, partially proving the Perdomo Conjecture.

## **Mean curvature flow through neck-singularities**

Robert Haslhofer (University of Toronto)

**Abstract:** In this talk, I will explain our recent work showing that mean curvature flow through neck-singularities is unique. The key is a classification result for ancient asymptotically cylindrical flows that describes all possible blowup limits near a neck-singularity. In particular, this confirms Ilmanen's mean-convex neighborhood conjecture, and more precisely gives a canonical neighborhood theorem for neck-singularities. Furthermore, assuming the multiplicity-one conjecture, we conclude that for embedded two-spheres mean curvature flow through singularities is well-posed. The two-dimensional case is joint work with Choi and Hershkovits, and the higher-dimensional case is joint with Choi, Hershkovits and White.

## **The Minkowski problem in gaussian probability space**

Yong Huang (Hunan University)

**Abstract:** In this talk, we will discuss the Minkowski problem in Gaussian probability space. In the solving process, the Aleksandrov's variational method leads to an existence result with Lagrange multipliers. To avoid it, we provide existence results smooth solution with the degree theory of PDE, then an approximation argument is deployed to obtain the weak solution. This is a joint work with Xi dongmeng (Shanghai University) and Zhao Yiming(MIT).

## **Gromov-Hausdorff limit spaces of manifolds and some applications**

Wenshuai Jiang (Zhejiang University)

**Abstract:** In this talk, we will review some recent development of the study of Gromov-Hausdorff limit space. Some related applications would be also discussed.

## **Dirac-harmonic maps**

Jürgen Jost (Max Plank Institute)

**Abstract:** Dirac-harmonic maps are a mathematical version of the nonlinear supersymmetric sigma model of quantum field theory. They couple a map from a Riemann surface to some Riemannian manifold with a spinor along that map. For a critical point of the action functional, a second order harmonic map type equation is coupled with a first order nonlinear Dirac equation. The resulting system is very difficult, but conformal invariance provides some powerful tools for the regularity of solutions. Since the action functional is not bounded from below, solutions cannot be found by minimization. Therefore, we have introduced and analyzed a mixed elliptic-parabolic system as a new tool. The techniques developed in this project are of wider interest in the geometric analysis of conformally invariant variational problems.

## **The YTD conjecture for generalized Kahler-Ricci solitons and an application**

Chi Li (Rutgers University)

**Abstract:** I will talk about generalized Kahler-Ricci soliton equations on log Fano pairs, which generalize the Kahler-Einstein and Kahler-Ricci soliton equations on Fano manifolds. Recent works by Apostolov et al. establish an interesting connection of generalized soliton equation with Sasaki-Einstein metrics or equivalently Ricci-flat Kahler cone metrics. I will prove a version of the Yau-Tian-Donaldson (YTD) conjecture for generalized solitons and explain the application to Ricci-flat Kähler cone metrics. This is based on a joint work with Jiyuan Han.

## **Geometric inequalities for static convex domains in hyperbolic space**

Haizhong Li (Tsinghua University)

**Abstract:** We introduce two new kinds of locally constrained inverse curvature flows in hyperbolic space. We prove that if the initial hypersurface is static convex, then the solution of these flows becomes strictly static convex for  $t > 0$ . Using the static convexity of the flow hypersurfaces, we prove some new family of geometric inequalities for such hypersurfaces in hyperbolic space. This work is joint with Yingxiang Hu.

## **Symplectic Critical Surfaces and its related evolution**

Jiayu Li (University of Science and Technology of China)

**Abstract:** In this talk, we introduce new functionals to study the existence of holomorphic curves in Kähler surfaces. We will talk the properties of the critical surfaces of the functionals, and the gradient flow of the functionals.

## **Boundary Regularity and Stability of Spaces with Lower Ricci Bounds**

Aaron Naber (Northwestern University)

**Abstract:** Consider a noncollapsed metric measure space  $(X, d, H^n)$  with Ricci curvature bounded from below, precisely a  $RCD(n, K)$  space. Together with Brue and Semola we show that the boundary  $\partial X$  is an  $(n - 1)$  manifold with a rectifiable structure, away from a  $(n - 2)$  dimensional subset. In particular this answers some conjectures of Kapovitch/Mondino. The key new result is an eps-regularity theorem which says that if a ball is GH close to a half space, then it must be homeomorphic to a half space. The result requires a very new set of tools than dealing with the regular set, and gives a new and analytic proof that limits of manifolds  $M_i \rightarrow X$  have no codimension 1 singularities. In the case of limits of singular spaces  $X_i \rightarrow X$ , we

additionally prove a volume convergence for the boundary. Namely, we show that  $\partial X_i$  equipped with the  $n-1$  hausdorff measure converges to  $\partial X$  equipped with the  $(n-1)$  hausdorff measure.

## Positive mass theorems of ALF and ALG manifolds

Yuguang Shi (Peking University)

**Abstract:** In this talk, we will prove positive mass theorems for ALF and ALG manifolds with model spaces  $R^{n-1} \times S^1$  and  $R^{n-2} \times T^2$  respectively in dimensions no greater than 7. Different from the compatibility condition for spin structure in Theorem 2 of V. Minerbe's paper A mass for ALF manifolds, Comm. Math. Phys. 289 (2009), no. 3, 925–955 we show that some type of incompressible condition for  $S^1$  and  $T^2$  is enough to guarantee the nonnegativity of the mass. As in the asymptotically flat case, we reduce the desired positive mass theorems to those ones concerning non-existence of positive scalar curvature metrics on closed manifolds coming from generalize surgery to  $n$ -torus. Finally, we investigate certain fill-in problems and obtain an optimal bound for total mean curvature of admissible fill-ins for flat product 2-torus  $S^1(l_1) \times S^1(l_2)$  This talk is based on the paper joint with my Ph.D. students Peng Liu and Jintian Zhu, here is the link of the paper:<http://arxiv.org/abs/2103.11289>

## Moduli space of Kähler-Einstein metrics of negative scalar curvature

Jian Song (Rutgers University)

**Abstract:** Let  $K(n, V)$  be the space of  $n$ -dimensional compact Kähler-Einstein manifolds with negative scalar curvature and volume bounded above by  $V$ . We prove that any sequence in  $K(n, V)$  converges in pointed Gromov-Hausdorff topology to a finite union of complete Kähler-Einstein metric spaces without loss of volume, which is biholomorphic to an algebraic semi-log canonical model with its non-log terminal

locus removed. We further show that the Weil-Petersson metric extends uniquely to a Kähler current with continuous local potentials on the KSB compactification of the moduli space canonically polarized manifolds. In particular, the Weil-Petersson volume of the KSB moduli space is finite.

## **Partial $C^0$ -estimate and Hamilton-Tian conjecture**

Feng Wang (Zhejiang University)

**Abstract:** Hamilton-Tian conjecture says that the Kahler-Ricci flow on Fano manifolds converges to a limit space admitting Kahler-Ricci soliton outside the singularity of dimension 4. This conjecture has been proved by Chen-Wang and Bamler. All their proofs depend on the metric geometry. Using Liu-Szekelyhidi's work on partial  $C^0$  estimate, we will give another proof of Hamilton-Tian conjecture. Some related questions will also be discussed.

## **The relative isoperimetric inequality**

Guofang Wang (University of Freiburg)

**Abstract:** In this talk, we discuss the relative isoperimetric inequality, its generalization and applications. The talk is based on a joint work with Lei Liu and Liangjun Weng.

## **TBA**

Paul Yang (Princeton University)

## **Kähler-Ricci flow on smooth minimal models**

Zhenlei Zhang (Capital Normal University)

**Abstract:** It is a survey talk on the convergence of Kähler-Ricci flow on smooth manifolds with nef canonical line bundles.

## **On singular cscK metrics**

Kai Zheng (Tongji University)

**Abstract:** In this talk, we first will present recent progress on the Yau-Tian-Donaldson conjecture in the logarithmic setting. Then we will carry out a comparison of different K-stability notions and show various results when the underlying class is merely big.

## **Existence of constant mean curvature surfaces**

Xin Zhou (Cornell University)

**Abstract:** Constant Mean Curvature (CMC) surfaces constitute a classical subject in Differential Geometry and are mathematical models in many disciplines of science. In this talk, I will present a recent work on the existence of CMC 2-spheres in an arbitrary Riemannian 3-sphere. This is a joint work with Da Rong Cheng.

## **Geometric Analysis on Metric Spaces**

Xiping Zhu (Sun Yat-sen University)

**Abstract:** The concept of sectional curvature (or Ricci curvature, respectively) bounded from below was successfully extended to certain metric spaces by Alexandrov (or Lott-Sturm-Villani, respectively). In this talk I will discuss the compatibility of

these two senses and then study some elliptic partial differential equations on metric spaces.

## **Application of the partial $C^0$ -estimate in Kähler-Ricci flow**

Xiaohua Zhu (Peking University)

**Abstract:** The partial  $C^0$ -estimate was first introduced by Tian in his pioneering work on KE metrics on Fano surfaces in 1990. This estimate builds a bridge between Kähler geometry and Riemannian geometry so that a (polarized) Kähler metric can be possibly compared by an induced metric of complex submanifold in the projective space. In this talk, I will discuss the application of this estimate in the study of global convergence of Kähler-Ricci flow on Fano manifolds as well as the local regularity of flow.