High order conservative Lagrangian schemes for radiative hydrodynamics equations

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Abstract

Radiation hydrodynamics describes the interactions between matter and radiation which affect the thermodynamic states and the dynamic flow characteristics of the matter-radiation system. Its application areas are mainly in high-temperature hydrodynamics, including astrophysics, reentry vehicles fusion physics and inertial confinement fusion. In this talk, we will discuss the methodology to construct fully explicit and implicit-explicit (IMEX) high order Lagrangian schemes solving the one-dimensional radiative hydrodynamics equations in the equilibrium diffusion limit respectively, which can be used to simulate multi-material problems with the coupling of radiation and hydrodynamics. The schemes can maintain conservation and uniformly high order accuracy both in space and time. The issue of positivity-preserving for the explicit high order Lagrangian scheme is also discussed. Various numerical tests for the high order Lagrangian schemes are provided to demonstrate the desired properties of the schemes such as high order accuracy, non-oscillation, and positivity-preserving.

High order finite difference WENO schemes for ideal magnetohydrodynamics on curvilinear meshes

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Abstract

In this talk, I will present a high order finite difference weighted essentially non-oscillatory (WENO) scheme for the ideal magnetohydrodynamic (MHD) equations on curvilinear meshes. In order to control the divergence error of magnetic filed, the constrained transport framework will be considered. Instead of

solving the magnetic field directly, a magnetic potential is introduced. We will simultaneously evolve the ideal MHD equations and magnetic potential equations alongside each other, and set the magnetic field to be the (discrete) curl of the magnetic potential after each time step. However, the free-stream property is difficult to fulfill if we use standard WENO schemes for both ideal MHD equations and the potential equation. Here, we will design finite difference WENO schemes to solve both equations. Theoretical derivation and numerical results show the proposed numerical scheme can preserve the free-stream property on generalized coordinate systems, hence giving much better performance than the standard finite difference WENO schemes for such problems.

A Low Rank Tensor Representation Based on WENO Schemes for Nonlinear Vlasov Dynamics

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Abstract

We present a low-rank tensor approach for solving the Vlasov equation. Among many existing challenges for Vlasov simulations (e.g. multi-scale features, nonlinearity, formation of filamentation structures), the curse of dimensionality and the associated huge computational cost have been a long-standing key obstacle for realistic high-dimensional simulations. In this work we propose to overcome the curse of dimensionality by dynamically and adaptively exploring a low-rank tensor representation of Vlasov solutions in a general high-dimensional setting. In particular, we develop two different approaches: one is to directly solve the unknown function, and the other is to solve the underlying flow map, aiming to obtain a low-rank approximation with optimal complexity. The performance of both proposed algorithms are benchmarked for standard Vlasov-Poisson/Maxwell test problems.

Convolution Neural Network Based Shock Detector for Numerical Solution of Conservation Laws and Hybrid WENO Method

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Abstract

In this presentation, we present a hybrid finite difference WENO method for hyperbolic conservation laws with a convolution neural network (CNN) based shock detector. The CNN detector is trained offline with synthetic data. The training data are generated using randomly constructed piecewise functions, which are then processed using randomized linear advection solver to count for the cases of numerical errors in practice. The detector is then paired with high-order WENO numerical solvers. One- and two-dimensional numerical examples on scalar and Euler equations are provided to demonstrate the performance of the proposed method. Comparison with the classical shock detectors illustrates that the neural-network based shock detector is able to provide cleaner and sharper signals without a problem-dependent parameter.

Bound preserving flux limiters and total variation stability

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Abstract

In this talk, we will discuss a family of bound preserving flux limiters for high order finite difference WENO methods solving hyperbolic conservation laws. This family of flux limiters are generalized from the flux-corrected transport method to high order methods solving scalar conservation laws by Boris and Book. We will discuss the algorithm for preserving a global maximum principle and its application to obtain total variation stability for high order methods solving one dimensional scalar conservation law.

复杂问题数值模拟和理论分析

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摘要

复杂问题包括复杂物理问题和复杂几何问题，本报告指的复杂物理问题主要是气动声学，由于其具有强非定常和多尺度特性，对数值方法的要求特别高，需要具有高精度、高分辨率和低耗散特性，对超声速流动，还需要格式具有一定的捕捉激波能力。对复杂几何问题是指高机动旋转飞行器和包括开舱门的大尺度空腔问题，对复杂几何问题的高精度模拟，除了上述要求之外，需要格式具有很好的鲁棒性。

本报告主要介绍我们针对这两类复杂问题设计的高阶精度高分辨率计算方法以及从简单模型问题开始到具有一定工程实际的较复杂的问题的数值模拟研究。针对气动噪声产生机理和复杂几何流动多尺度涡产生演化过程，我们发展的从 Lagrangian 框架的分析理论。

A positivity-preserving HWENO scheme for compressible Navier-Stokes equations

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Abstract

The HWENO scheme have the advantage of smaller stencil than the conventional WENO scheme. However, HWENO scheme is usually less robust. We render the HWENO scheme stable for the compressible Navier-Stokes equations by preserving positivity of density and pressure through a positivity-preserving diffusion flux and a positivity-preserving limiter.

High order WENO fast sweeping schemes for solving steady state problems of hyperbolic PDEs

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Abstract

Fast sweeping methods are a class of efficient iterative methods for solving steady state problems of hyperbolic PDEs. They utilize alternating sweeping strategy to cover a family of characteristics in a certain direction simultaneously in each sweeping order. Coupled with the Gauss-Seidel iterations, these methods can achieve a fast convergence speed for computations of steady state solutions of hyperbolic PDEs. In this talk, I shall review our studies on developing high order accurate WENO fast sweeping schemes for solving steady state problems of Hamilton-Jacobi equations and hyperbolic conservation laws. Recent results on improving the convergence of the fast sweeping WENO methods will be discussed.

A WENO-based stochastic Galerkin scheme for ideal MHD equations with random inputs

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Abstract

In this talk, we investigate the ideal magnetohydrodynamic (MHD) equations with random inputs based on generalized polynomial chaos (gPC) stochastic Galerkin approximation. A special treatment with symmetrization is carried out for the gPC stochastic Galerkin method so that the resulting deterministic gPC Galerkin system is provably symmetric hyperbolic in the spatially one-dimensional case. We discretize the hyperbolic gPC Galerkin system with a high-order path-conservative finite volume weighted essentially non-oscillatory scheme in space and a third-order total variation diminishing Runge-Kutta method in time. The method is also extended to two spatial dimensions via the operator splitting technique. Several numerical examples are provided to illustrate the accuracy and effectiveness of the numerical scheme.

A new type of multi-resolution WENO schemes with increasingly higher order of accuracy for hyperbolic conservation laws

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Abstract

In this presentation, a new type of high-order finite difference and finite volume multiresolution weighted essentially non-oscillatory (WENO) schemes is presented for solving hyperbolic conservation laws. We only use the information defined on a hierarchy of nested central spatial stencils and do not introduce any equivalent multiresolution representation. These new WENO schemes use the same large stencils as the classical WENO schemes, could obtain the optimal order of accuracy in smooth regions, and could simultaneously suppress spurious oscillations near discontinuities. The linear weights of such WENO schemes can be any positive numbers on the condition that their sum equals one. This is the first time that a series of unequal-sized hierarchical central spatial stencils are used in designing high-order finite difference and finite volume WENO schemes. These new WENO schemes are simple to construct and can be easily implemented to arbitrary high order of accuracy and in higher dimensions. Benchmark examples are given to demonstrate the robustness and good performance of these new WENO schemes.