**Course information**

**Course 1: The existence of complements for Fano type varieties**

**Lecturer:** Jingjun Han

**Assistant:** Guodu Chen

**Abstract:** The theory of complements was introduced by Shokurov when he proved the existence of log flips for threefolds. It plays an important role in the study of Fano varieties, including the boundedness of Fano varieties, and K-stability in algebraic geometry. In this series of lectures, I will introduce some recent progress on the existence of complements for Fano type varieties as well as some applications of the theory of complements.

**References:**
V. V. Shokurov. Three-dimensional log perestroikas. Izv. Ross. Akad. Nauk Ser. Mat., 56(1):105--203, 1992.
V. V. Shokurov. Complements on surfaces. J. Math. Sci. (New York), 102(2):3876--3932, 2000.
Yu. G. Prokhorov and V. V. Shokurov. The first fundamental theorem on complements: from global to local. Izv. Ross. Akad. Nauk Ser. Mat., 65(6):99--128, 2001.
Yu. G. Prokhorov and V. V. Shokurov. Towards the second main theorem on complements. J. Algebraic Geom., 18(1):151--199, 2009.
Caucher Birkar. Singularities of linear systems and boundedness of Fano varieties. arXiv:1609.05543, 2016.
Caucher Birkar. Anti-pluricanonical systems on Fano varieties. Ann. of Math. (2), 190(2):345--463, 2019
Jingjun Han, Jihao Liu, and V. V. Shokurov. ACC for minimal log discrepancies of exceptional singularities. arXiv:1903.04338v2, 2019.

**Course 2: K-stability of Fano varieties**

**Lecturers:** Chenyang Xu, Ziquan Zhuang

**Assistant:** Kai Huang

**Abstract:** K-stability is a notion in algebraic geometry that has close relation with the existence of Kähler-Einstein metric. In these lectures, we will survey the recent understanding of K-stability through birational geometry and valuation theory. In particular, we will discuss Fujita-Li's valuative criterion and a few general methods for checking K-stability of explicit Fano varieties.

**References:**

K-stability of Fano varieties: an algebro-geometric approach, Chenyang Xu, see http://math.mit.edu/~cyxu/Kstability.pdf

**Course 3: Construction of K-moduli spaces**

**Lecturers:** Harold Blum, Yuchen Liu

**Assistant:** Chuyu Zhou

**Lecture 1: Introduction to the K-moduli Conjecture (by Harold Blum)**
In this talk, I will survey recent progress on the K-moduli Conjecture, which   predicts that K-polystable Fano varieties of fixed dimension and volume are parametrized by a projective good moduli space. Specifically, I will discuss complications one encounters when constructing moduli spaces of Fano varieties and the steps needed to prove the above conjecture (many of which are now known). If time permits, I will also discuss a relevant result of Li-Wang-Xu on K-polystable degenerations.

**References:**
Blum, Xu: Uniqueness of K-polystable degenerations of Fano varieties. Ann. of Math. (2) 190 (2019), no. 2, 609–656.
Li, Wang, Xu: Algebraic of the metric tangent cones and equivariant K-stability. arXiv:1805.03393.

**Lecture 2: Existence of a separated good moduli space (by Harold Blum)**
A key step in proving the K-moduli conjecture is showing that the stack of K-semistable Fano varieties admits a separated good moduli space. In  this talk, I will discuss recent work of Alper-Blum-Halpern-Leistner-Xu in which such good moduli spaces are constructed. The key inputs in the proof are results concerning degenerations of K-semistable Fano varieties and a recent criterion for when an Artin stack admits a good moduli space.

**References:**
Blum, Xu: Uniqueness of K-polystable degenerations of Fano varieties. Ann. of Math. (2) 190 (2019), no. 2, 609–656.
Alper, Blum, Halpern-Leistner, Xu: Reductivity of the automorphism group of K-polystable Fano varieties. arXiv:1906.03122.
Alper, Halpern-Leistner, Heinloth. Existence of moduli spaces for algebraic stacks. arXiv: 1812.01128.

**Lecture 3: Openness of K-semistability (by Yuchen Liu)**
Openness is an essential part in the construction of K-moduli spaces. In this talk, I will explain the proof of openness of K-semistability by Blum-Liu-Xu. The main ingredients in the proofs are lower semi-continuity of stability thresholds due to Blum-Liu, and boundedness of complements by Birkar.
**References:**
Blum, Liu, Xu: Openness of K-semistability for Fano varieties. arXiv:1907.02408.
Xu: A minimizing valuation is quasi-monomial. Ann. of Math. (2) 191 (2020), no. 3, 1003--1030.
Blum, Liu: Openness of uniform K-stability in families of Q-Fano varieties. arXiv:1808.09070.

**Lecture 4: Explicit K-moduli spaces (by Yuchen Liu)**
In this talk, I will survey results on explicit K-moduli spaces including del Pezzo surfaces (due to Odaka-Spotti-Sun), and cubic threefolds (due to Liu-Xu). If we have time, I will also talk about wall crossing phenomena in K-moduli of log Fano pairs such as plane curves (due to Ascher-DeVleming-Liu). An essential ingredient is the use of normalized volumes to control singularities at the boundary of K-moduli spaces.

**References:**
Odaka, Spotti, Sun: Compact moduli spaces of del Pezzo surfaces and KählerEinstein metrics. J. Differential Geom., 102(1): 127--172, 2016.
Liu, Xu: K-stability of cubic threefolds. Duke Math. J. 168 (2019), no. 11, 2029-2073.
Ascher, DeVleming, Liu: Wall crossing for K-moduli spaces of plane curves. arXiv:1909.04576.
Li: Minimizing normalized volumes of valuations. Math. Z. 289 (2018), no. 1-2, 491--513.
Liu: The volume of singular Kähler-Einstein Fano varieties. Compos. Math. 154 (2018), no. 6, 1131--1158.